In 1924, Louis de Broglie:

**PARTICLES EXHIBIT WAVE-LIKE BEHAVIOR**

**Light**
- momentum
- wavelength

**Particles**
- momentum
- wavelength

“wave-particle duality”

**de Broglie relation:**

\[ \lambda = \frac{h}{p} \]

\[ p = m \cdot v \]
In 1927 diffraction experiment by Davisson and Germer

\[ \lambda = \frac{h}{p} \]

\( \lambda \) is high if \( p \) is low, but still not large even for electrons!

Grating of atomic dimensions: Crystal Structure

Diffraction of electrons from crystalline NiO verifying the de Broglie relation
There must be a **wave equation** that relates the spatial and time dependencies of the wave amplitude for the (wave-like) particles.

**Diffraction by a double slit**

**Single slit case:**

\[ \alpha \sin \theta = n \lambda \]

Minima

\[ n = \pm 1, \pm 2, \ldots \]
The double-slit diffraction experiment. Case 1 describes the outcome of the diffraction when one of the slits is blocked. Case 2 describes the outcome when both slits are open.

"Particle passing through both slits simultaneously" \( \Rightarrow \) interference pattern
. Arrival of each individual electron ⇒ a flash localized on the screen

"particle behavior"

. Many electrons needed to intensify signal
Considering many electrons:

"Each electron has a random phase angle with respect to every other electron."

⇒ Two electrons can never interfere with each other

Considering single electron:

⇒ A wave of a single electron is incident on both of the slits simultaneously ⇒ interference pattern
Classical physics

• It is not consistent for particles

Quantum physics

• The electron wavefunction is a superposition of wavefunctions going through the top slit and bottom slit, which is equivalent to saying that it goes through both slits.

• The act of measurement such as closing on slit changes the wavefunction.

In 1997, experiment with He atoms

$\psi(x) = \psi_t(x) + \psi_b(x)$

$\Rightarrow$ going through both slits
In 1910, Ernest Rutherford proposed that negative and positive charges are separated in an atom. Classical Theory suggests that electrons are constantly accelerating in their orbits, eventually emitting radiation and falling to the nucleus.
\[
\overline{v} = \frac{1}{\lambda} = R_H \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right), \quad n_2 \geq n_1 \text{ integer}
\]

\( \overline{v} \) is the Rydberg constant.
Angular Momentum of a Rotating Particle

\[ T = \frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 r^2 \]

\[ \Rightarrow T = \frac{1}{2} I \omega^2 \]

\[ I = m r^2 \]

\[ l = I \omega \] (like \( p = m v \) but for rot.)

\[ T = \frac{I^2}{2m} \] (like \( T = \frac{p^2}{2m} \))

Also \( l = mr^2 \omega = mvr \)
Niels Bohr

In 1911, proposed a model for hydrogen atom

A simple model of the Hydrogen atom in which an electron revolved around the nucleus in a circular orbit

\[ \frac{e^2}{4\pi \varepsilon_0 r^2} = \frac{m_e v^2}{r} + \lambda = \frac{\hbar}{\mathbf{p}} \]

\[ F = \frac{1}{4\pi \varepsilon_0} \frac{Q_1 Q_2}{r^2} \]

\[ 2\pi r = n \lambda \]

\[ r = r(n) \]
\[ 2\pi r = n\lambda \Rightarrow 2\pi r = n\frac{\hbar}{P} \]
\[ \Rightarrow m_e v r = n\frac{\hbar}{P} \]
\[ \frac{\hbar}{P} = \frac{\hbar}{2\pi l} \Rightarrow l = n\frac{\hbar}{2\pi} \quad \text{quantized} \]
\[ \Rightarrow r = \frac{\epsilon_0 \hbar^2 n^2}{10 m_e e^2} = \frac{4\pi \epsilon_0 \hbar^2 n^2}{m_e e^2} \]

Electron radii has certain discrete values

\[ E_{\text{total}} = E_K + E_P = \frac{1}{2} m_e v^2 - \frac{e^2}{4\pi \epsilon_0 r} \]
Eliminating \( \psi \):

\[
\hat{E}_{\text{total}} = \frac{1}{2} \left( \frac{e^2}{4\pi\varepsilon_0 r} \right) - \left( \frac{e^2}{4\pi\varepsilon_0 r} \right)
\]

\[
\hat{E}_{\text{total}} = -\frac{e^2}{8\pi\varepsilon_0 r}
\]

Eliminate \( r \):

\[
\Rightarrow \hat{E}_n = -\frac{m_e e^4}{8 \varepsilon_0^2 h^2 n^2}
\]

Ground state \( n = 1 \)

\( n = 2, 3 \)
\[ \Delta E = E_2 - E_1 = h \nu_2 - 1 \]

\[ \nu_2 - 1 = \frac{me^4}{8 \hbar^2 a^2} \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \]

\[ n_2 > n_1 \]

**Shortcomings**

- Only for hydrogen atom
- Contradicts Heisenberg uncertainty principle