In order to be in SHM, the restoring force must be proportional to the negative of the displacement. Here we have:

\[ F = -mg \sin \theta, \]

which is proportional to \( \sin \theta \) and not to \( \theta \) itself.

However, if the angle is small, \( \sin \theta \approx \theta \).
Therefore, for small angles, we have:

\[ F \approx -\frac{mg}{L} x, \]

where

\[ x = L \theta. \]

The period and frequency are:

\[ T = 2\pi \sqrt{\frac{L}{g}}, \]

\[ f = \frac{1}{2\pi} \sqrt{\frac{g}{L}}. \]
The Physical Pendulum

A physical pendulum is any real extended object that oscillates back and forth.

The torque about point O is:

\[ \tau = -mg \sin \theta. \]

Substituting into Newton’s second law gives:

\[ I \frac{d^2 \theta}{dt^2} = -mg \sin \theta. \]
For small angles, this becomes:

\[
\frac{d^2\theta}{dt^2} + \left(\frac{mgh}{I}\right)\theta = 0,
\]

which is the equation for SHM, with

\[
\theta = \theta_{\text{max}} \cos(\omega t + \phi),
\]

\[
T = 2\pi \sqrt{\frac{I}{mgh}}.
\]
Damped harmonic motion is harmonic motion with a frictional or drag force. If the damping is small, we can treat it as an “envelope” that modifies the undamped oscillation.

\[
F_{\text{damping}} = -bv, \\
ma = -kx - bv.
\]
\[ m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0. \]

If \( b \) is small, a solution of the form

\[ x = Ae^{-\gamma t} \cos \omega' t \]

will work, with

\[ \gamma = \frac{b}{2m}, \]

\[ \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}. \]
Forced Oscillations; Resonance

The equation of motion for a forced oscillator is:

\[ ma = -kx - bv + F_0 \cos \omega t. \]

The solution is:

\[ x = A_0 \sin(\omega t + \phi_0), \]

where

\[ A_0 = \frac{F_0}{m \sqrt{(\omega^2 - \omega_0^2)^2 + b^2 \omega^2 / m^2}} \]

and

\[ \phi_0 = \tan^{-1} \left( \frac{\omega_0^2 - \omega^2}{\omega (b/m)} \right). \]
The width of the resonant peak can be characterized by the $Q$ factor:

$$Q = \frac{m\omega_0}{b}.$$